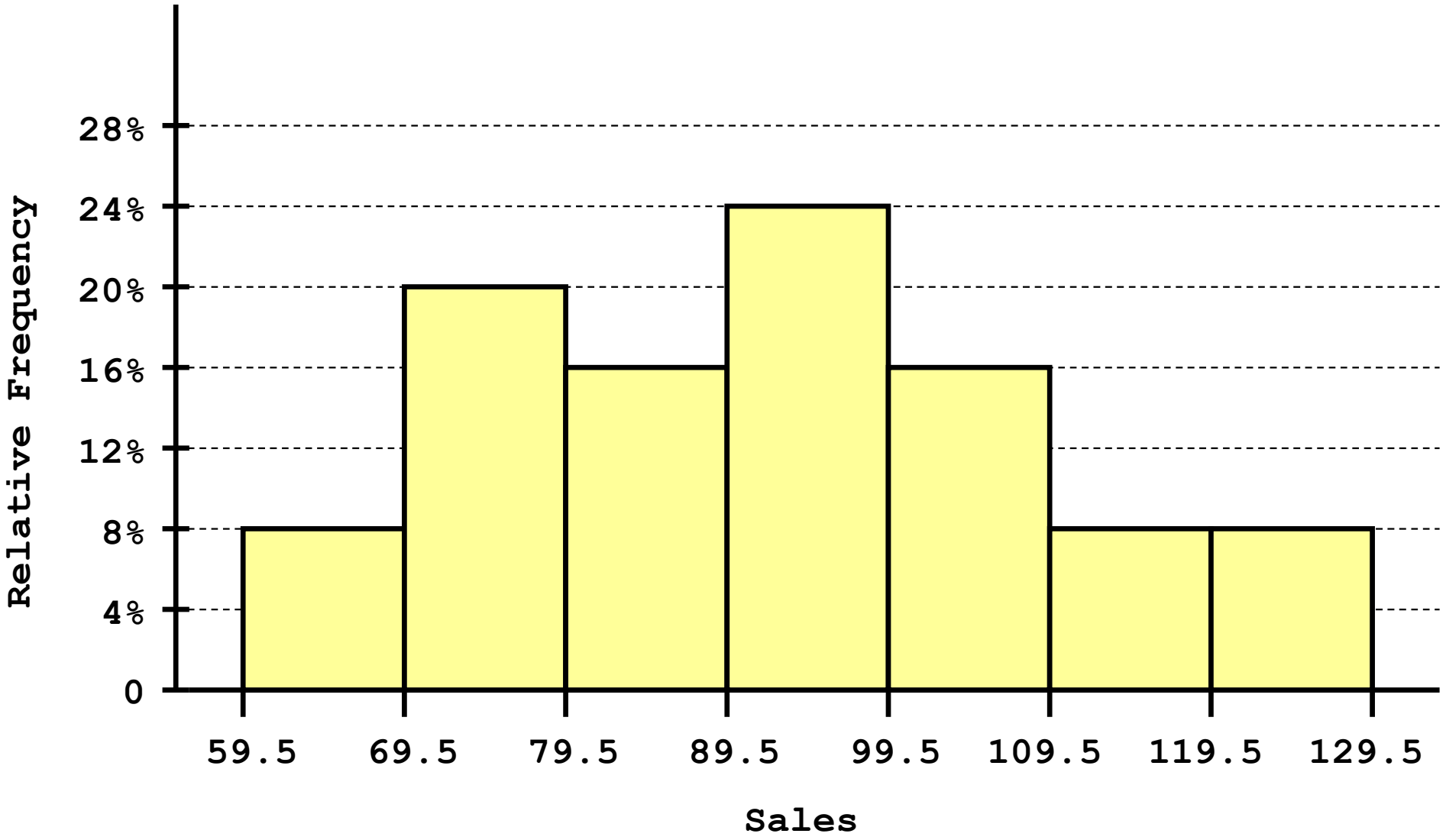
Confidence Intervals about a Population Mean (z only)

Point Estimate of a Population Mean

- The following are the invoice amounts for 25 invoices drawn at random from last quarter's sales data:

82	77	97	100	99
105	112	68	93	72
126	71	97	84	98
76	67	109	83	100
86	94	77	121	115

Point Estimate of a Population Mean



Point Estimate of a Population Mean

- A **point estimate** is a single value used to approximate a **population parameter**.
- The best point estimate of the population mean (μ) is the sample mean (x-bar).
- The best point estimate of the population standard deviation (σ) is the sample standard deviation (s).

Point Estimate of a Population Mean

- For the given data, n=25, x-bar = 92.4 and s=16.7.
- But 92.4 is only an estimate of μ. How sure are we that it is a "good" estimate? How "good" is it?
- What if we sampled n=250 invoices and found the same sample mean? We would intuitively have more confidence in the second statistic than in the first.
- But the problem with a point estimate is that we cannot assign a statistical **level of confidence** to it.

Interval Estimates

- We can, however, assign a level of confidence to an **interval estimate**.
- If you were asked to come up with a 95% confidence interval for the first case (x-bar = 92.4, n = 25), you might say you were 95% confident that the true mean is in the interval 92.4 ± 10.
- But in the second case (x-bar = 92.4, n=250), you might say you were 95% confident that the true mean is in the interval 92.4 ± 4.

(Numbers used above are "guesses" only, for illustrative purposes.)

CI for Population Mean (σ known)

- The formula for the **confidence interval** (CI) for a population mean is usually shown as:

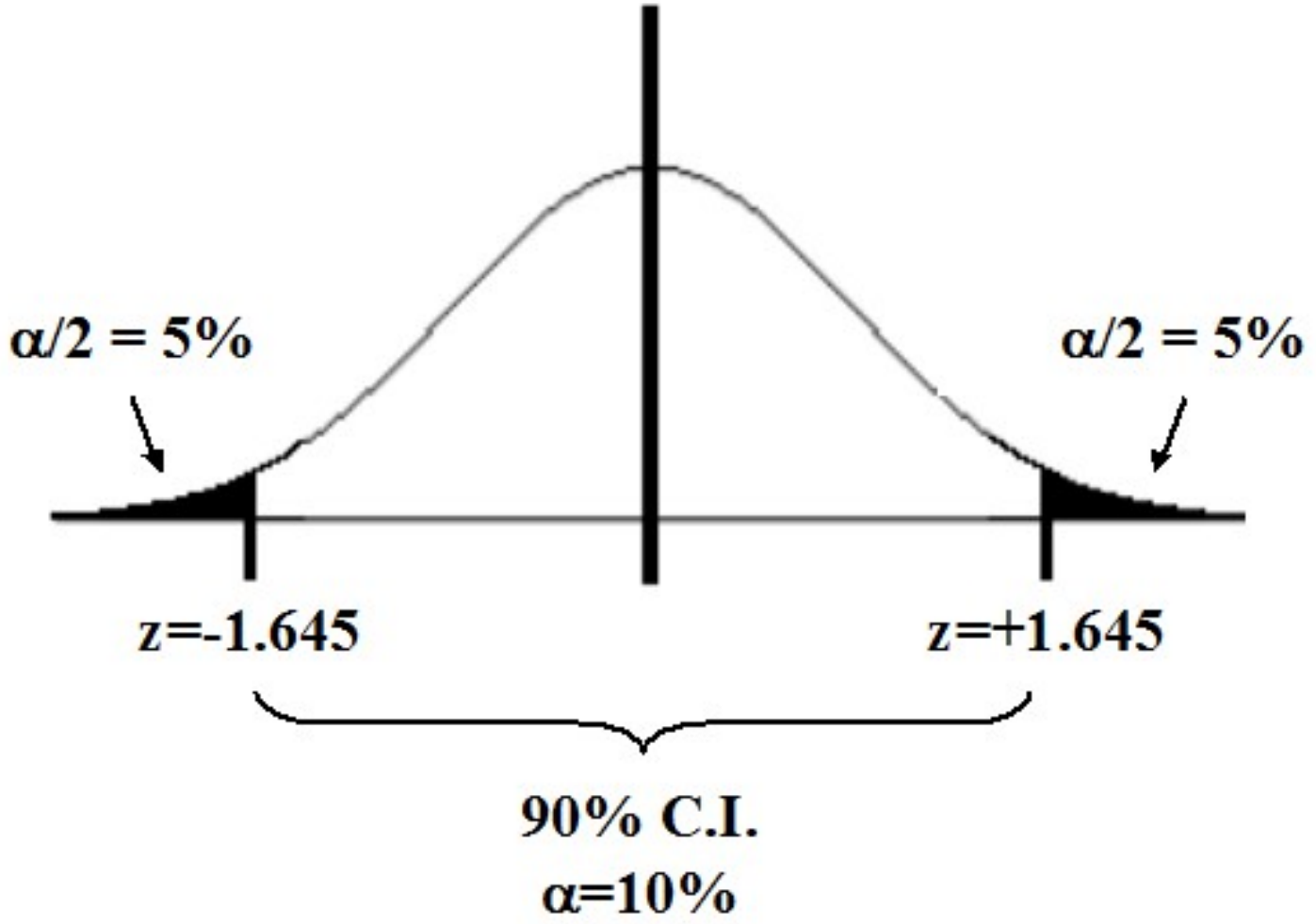
$$\mu = \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

- Or sometimes $\mu = \bar{x} \pm E$ where E is the **margin of error** and is calculated as:

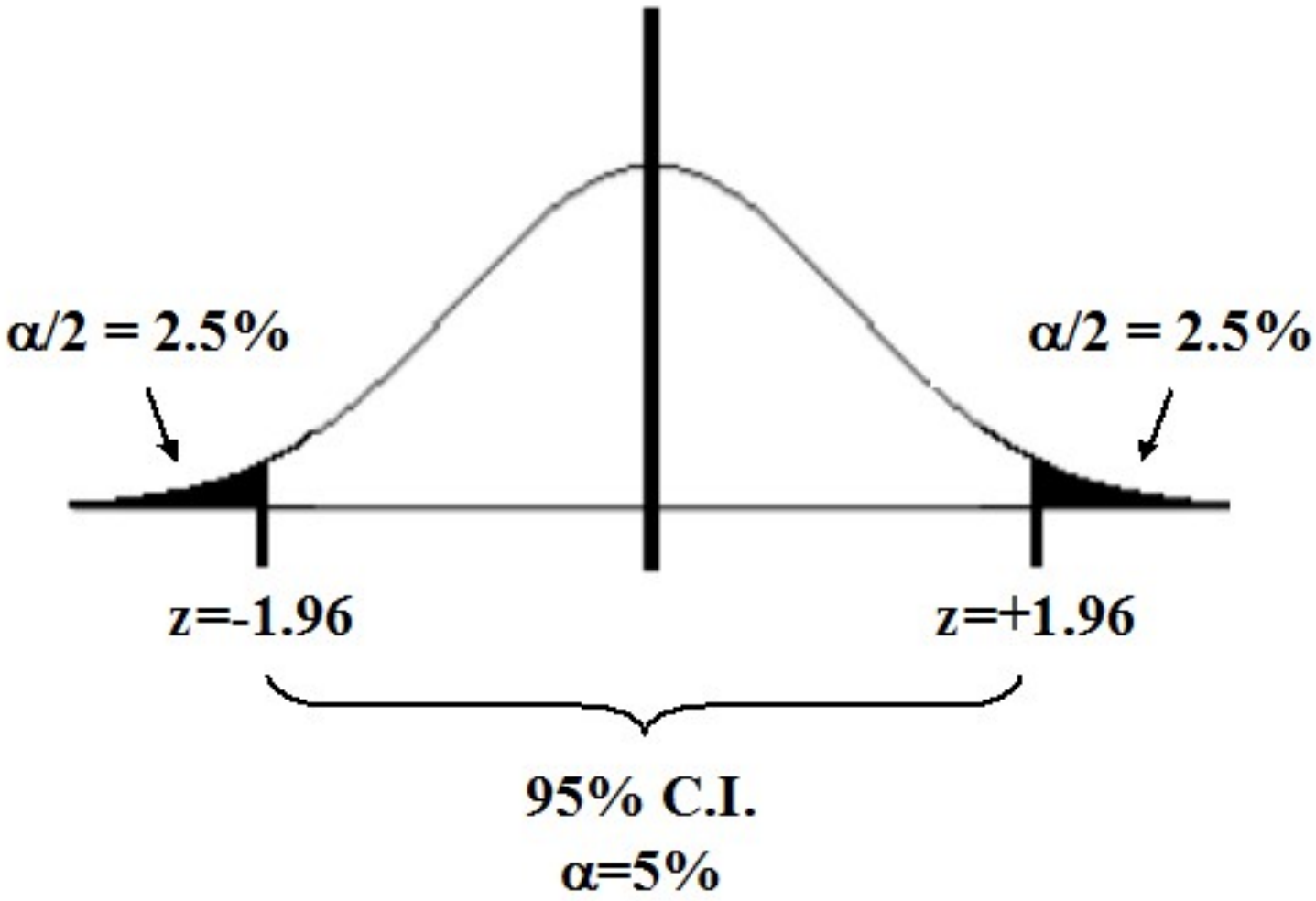
$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

- We use σ if known (only in stats textbooks!), otherwise we use s.

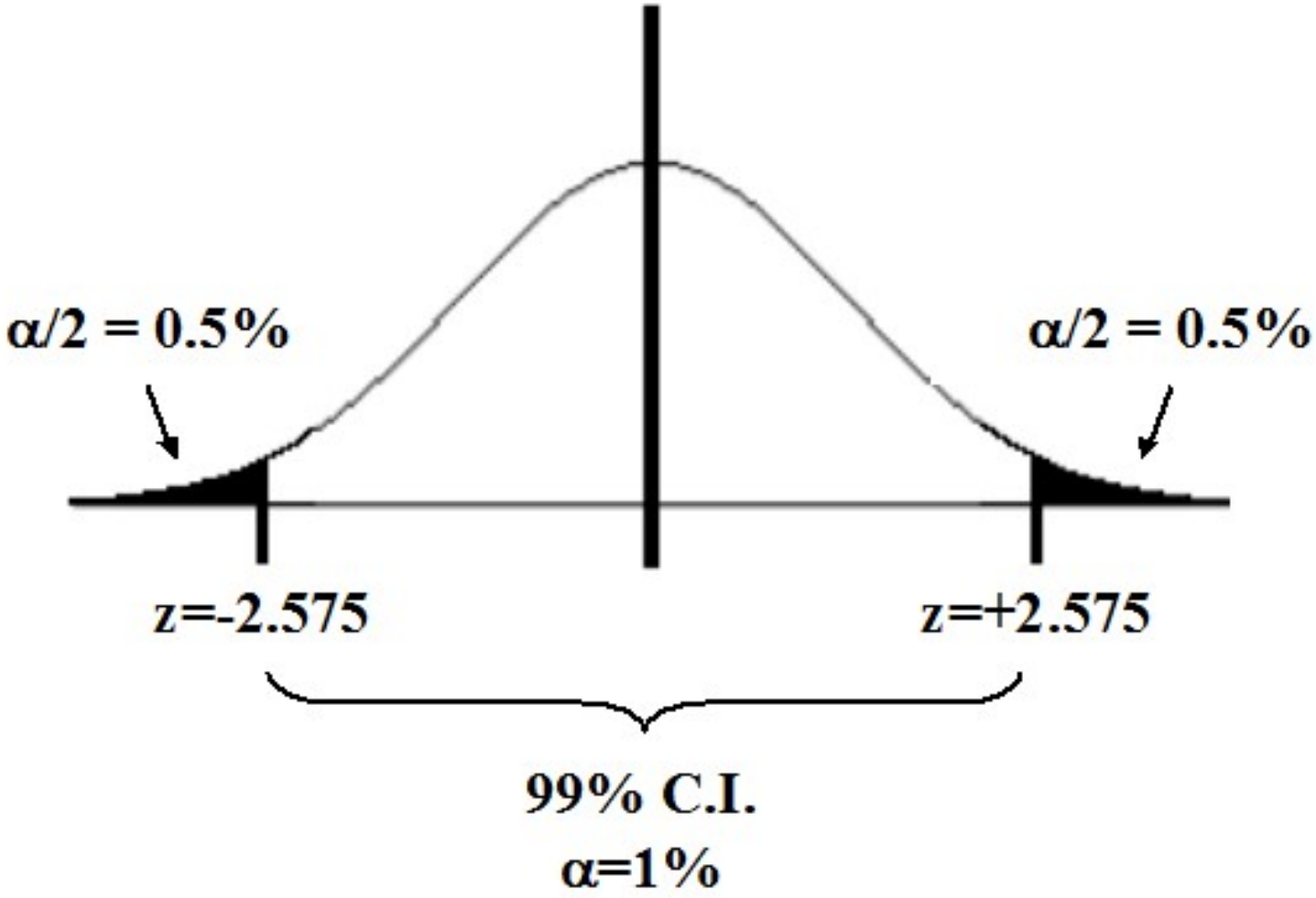
90% Confidence Interval



95% Confidence Interval



99% Confidence Interval



Calculating Confidence Intervals

Confidence Interval	Two-tailed T. O. H.	One-tailed T. O. H.
90% → z = 1.645	α = .01 → z = 2.575	α = .01 → z = 2.326
95% → z = 1.96	α = .05 → z = 1.96	α = .05 → z = 1.645
99% → z = 2.575	α = .10 → z = 1.645	α = .10 → z = 1.282

Together

- Find the 95% confidence interval for the mean invoice amount using the sample data: n=250, x-bar = 92.4. Assume σ is known to be 16.7.
- Solution:

$$\mu = \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 92.4 \pm 1.96 \frac{16.7}{\sqrt{250}}$$

$$= 92.4 \pm 2.1$$

$$= (90.3, 94.5)$$

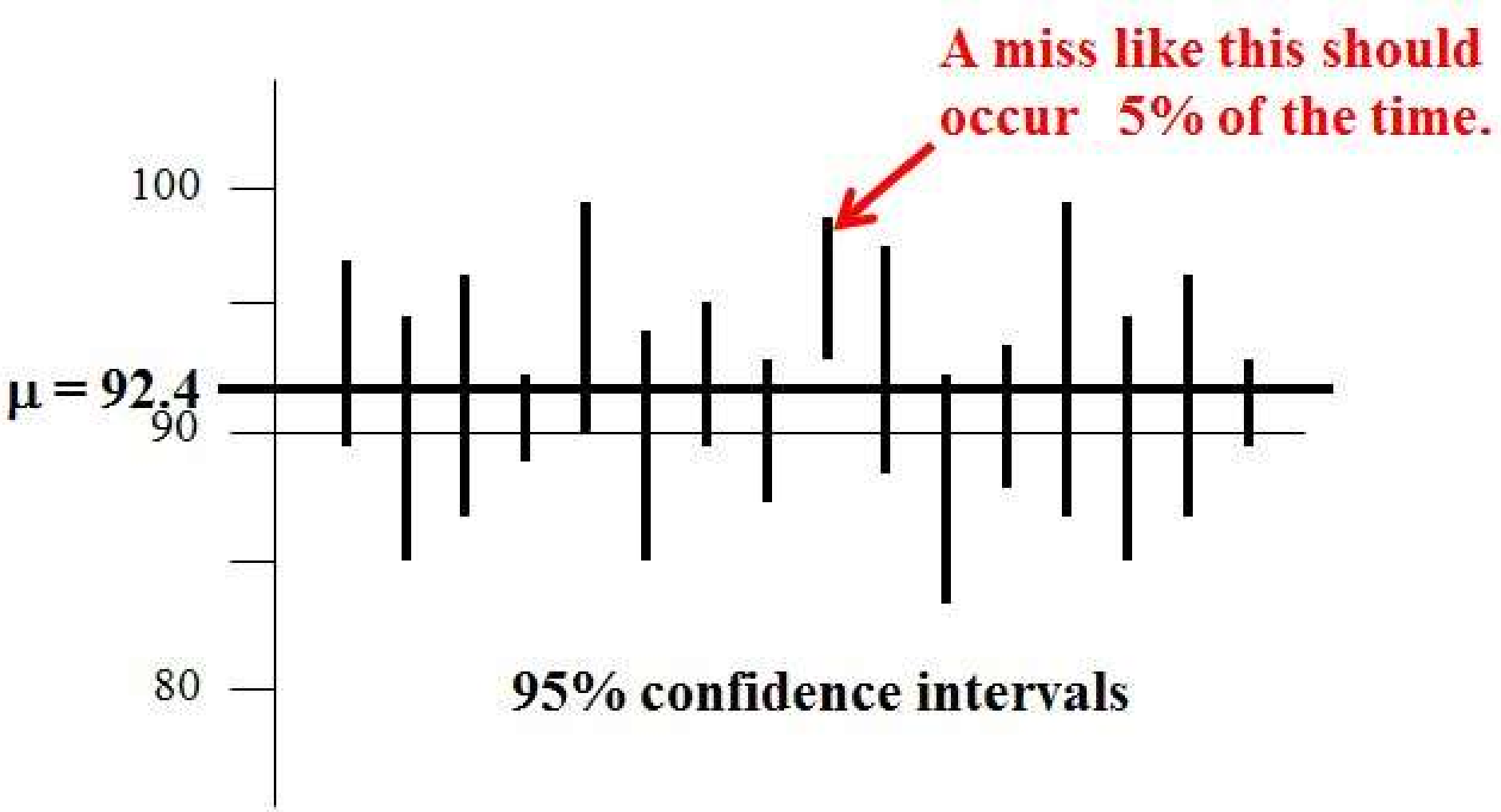
Interpretation

So what does it mean?

Wrong: We are 95% confident that the population mean is between 90.3 and 94.5.

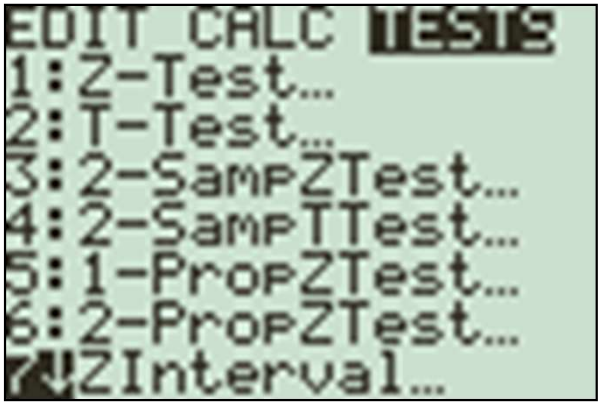
Correct: If the sampling process were repeated many times, and the interval calculated each time, 95% of those intervals would capture the true mean.

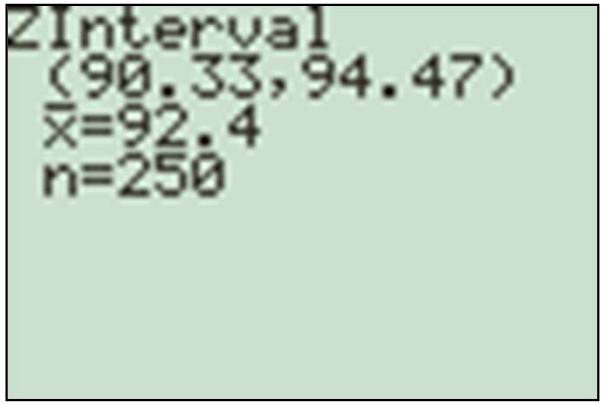
Interpretation



Using the TI-83 Plus

- Press [STAT] [TESTS] [ZInterval]
- Choose Stats if summary statistics are being used.
- Supply data. Note: σ is required. That's a big hint!





Together

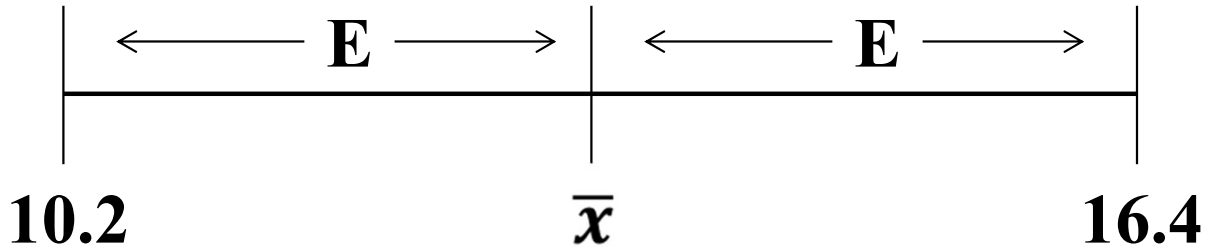
Find the 99% confidence interval for the population mean μ of the gambling losses suffered by Packers fans following the infamous substitute referee debacle of September 24, 2012 given n = 40 and x-bar = $189. Assume σ is known to be $87.

Aside from mentioning the Packers, what's wrong with this question?

Margin of Error

Given a confidence interval of [10.2, 16.4].

- What is the mean? (Answer: 13.3)
- What is the margin of error? (Answer: 3.1)



- What is the margin of error for the previous problem?

Together

Find the margin of error: