Tests of Hypotheses

Objectives

At the end of this section you should be able to perform tests of hypothesis for a population mean and a population proportion.

Specifically, you should be able to:

- utilize the eight steps in performing a test of hypothesis
- perform a test of hypothesis about a mean
- perform a test of hypothesis about a proportion

Claims

We will conduct tests of hypotheses for the following claims:

- A fast food vendor claims its macho burrito weighs over a half pound. (n=18)
- A test prep company claims a mean improvement of 50 points on a standardized test. (n=40)
- A pharmaceutical company claims its drug is effective on 80% of patients. (n=200)

We will, in each case, take the position of a competitor who disputes the stated claim.

Tests of Hypotheses -- Eight Steps

No new stats here, just a formalism with eight steps:

1. State the hypothesis
2. Identify the test statistic to be used
3. Determine the alpha to be used
4. Identify the critical value(s) / rejection region
5. Draw the sample
6. Calculate the observed value of the test statistic
7. State the conclusion
8. Find the p-value.

Note: hypothesis = singular, hypotheses = plural.

Tests of Hypotheses -- Eight Steps

1. State the hypothesis

- Always the most confusing part for students!
- Example: H_0: $\mu = 100$ vs. H_1: $\mu \neq 100$
- More confusing in stats books than in real life
- H_0 is null hypothesis, H_1 is alternate hypothesis
- Equality goes with the null hypothesis
- Generally, the statement which is being disputed is the null hypothesis, while that which you hope to establish (but cannot prove!) is the alternate hypothesis.

(We will use the macho burrito claim to illustrate each step.)

Tests of Hypotheses -- Eight Steps

2. Identify the test statistic to be used

- Examples:

$$\cancel{z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}}$$

$$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

For tests of μ when σ is known, which is almost never!

For tests of μ when σ unknown. Use df = n – 1.

For tests of p. Requires $np_0 \geq 15$ and $nq_0 \geq 15$.

Tests of Hypotheses -- Eight Steps

3. Determine the alpha to be used

- Usually .01, .05, or .10.
- If one is not given, then use .05.
- This is the probability you are willing to accept that you will reject the null hypothesis when in fact it is true.

Tests of Hypotheses -- Eight Steps

4. Identify the critical value(s) / rejection region

- Draw a picture!
- If H_1 contains $\neq$, then shade two tails.
- If H_1 contains **<** or **>**, then shade the tail that H_1 is pointing too.
- That's the rejection region.
- Now write it as a **rejection rule**.
- Example: Reject H_0 if $t < -1.740$

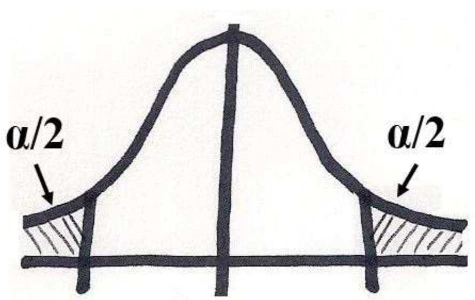
Rejection Region

Given: The average weight of a third grade girl in 2000 was 78 pounds.

Research Question

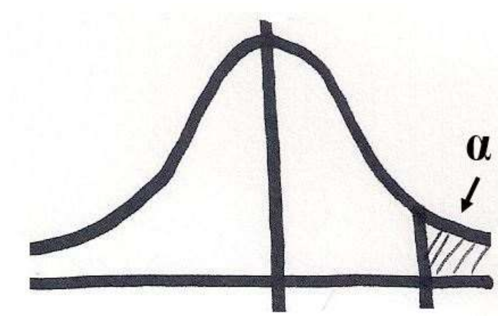
Has the average weight changed?

$H_0: \mu = 78$ vs. $H_1: \mu \neq 78$



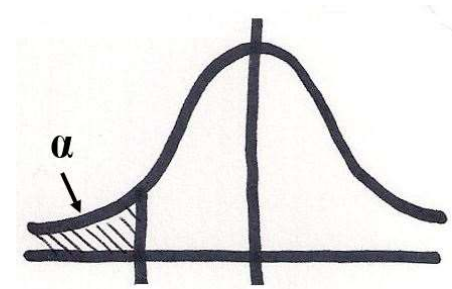
Has the average weight increased?

$H_0: \mu = 78$ vs. $H_1: \mu > 78$



Has the average weight decreased?

$H_0: \mu = 78$ vs. $H_1: \mu < 78$



t table (extract)

	α				
	.005 (1 tail)	.01 (1 tail)	.025 (1 tail)	.05 (1 tail)	.10 (1 tail)
df	.01 (2 tails)	.02 (2 tails)	.05 (2 tails)	.10 (2 tails)	.20 (2 tails)
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325

Tests of Hypotheses -- Eight Steps

- If you are performing a test of hypothesis about a population mean with unknown population standard deviation (almost always) then you need the equivalent of the INVT function, which is found on the TI-84 but not on the TI-83.
- An excellent video which will show you how to program the INVT function into your TI-83 can be found on YouTube at http://www.youtube.com/watch?v=5Ft5eZVJtPk

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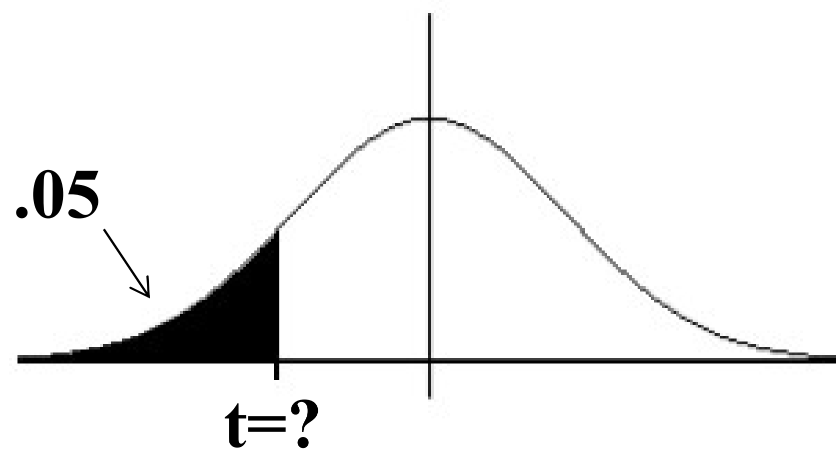
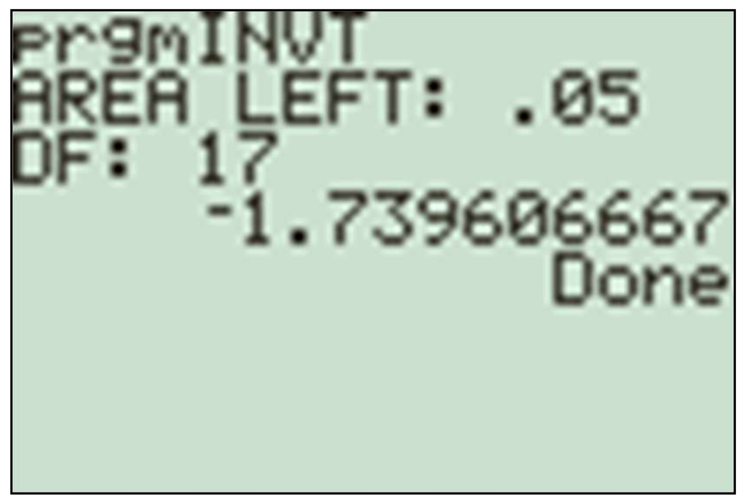
Tests of Hypotheses -- Eight Steps

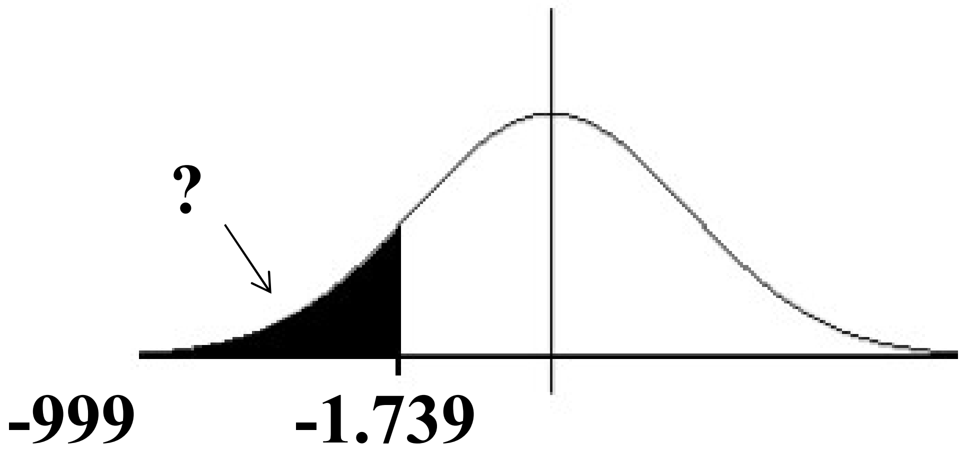
- Using INVT program...

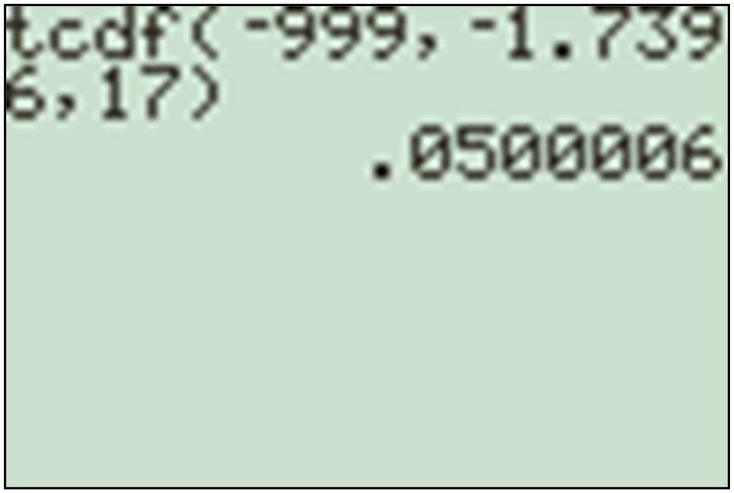




Tests of Hypotheses -- Eight Steps

- Using TCDF (built in)...





tcdf(left, right, df)

Tests of Hypotheses -- Eight Steps

Confidence Interval	Two-tailed T. O. H.	One-tailed T. O. H.
90% → z = 1.645	α = .01 → z = 2.575	α = .01 → z = 2.326
95% → z = 1.96	α = .05 → z = 1.96	α = .05 → z = 1.645
99% → z = 2.575	α = .10 → z = 1.645	α = .10 → z = 1.282

Use these when performing a test of hypothesis about a population proportion. Or use invNorm.

Tests of Hypotheses -- Eight Steps

5. Draw the sample

- Example: x-bar = ____, s = ____, or p-hat = ____
- If you are doing honest research, now is the time to draw the sample.
- This point is usually lost in the problems given in statistics books.

Assume x-bar = 7.8 ounces, s = 0.4 ounces.
Answer = -2.12

Tests of Hypotheses -- Eight Steps

6. **Calculate the observed value of the test statistic**

- Refer back to step 2. Do it.

Tests of Hypotheses -- Eight Steps

7. State the conclusion

- Refer back to rejection rule in step 4.
- Example: "Reject H_0" or "Cannot reject H_0"
- So what does it mean "in English?

Tests of Hypotheses -- Eight Steps

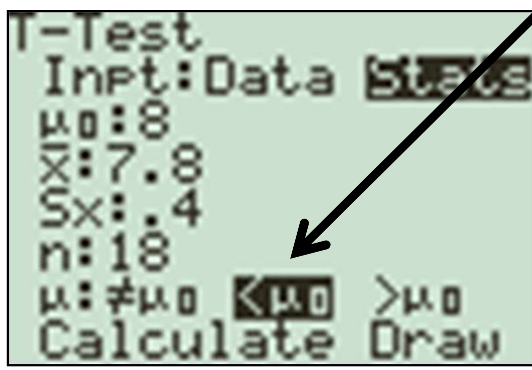
8. Find the p-value.

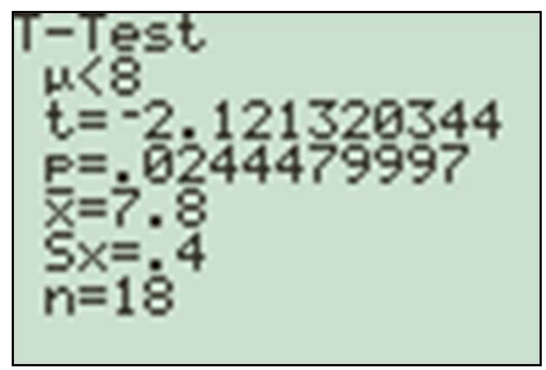
- Not to be confused with the p as in proportion!
- It is the probability of observing a test statistic "this extreme or more so" if H_0 were true.
- p-value is required for APA format
- Example: Never "$p = 0$". Use "$p < .001$"
- Use tcdf (for t) or normalcdf (for z) function.
- For two-tailed tests, multiply by two.
- Gut check: Reject H_0 when p-value < α

Testing Claims about a Population Mean μ

TI-83/84 PLUS: If using a TI-83/84 Plus calculator, press **STAT**, then selected **TESTS** and choose the second option, **T-Test**. You can use the original data **(Data)** or the summary statistics (**Stats**) by providing the entries indicated in the window display. The first three items of the TI-83/84 Plus calculator results will include the alternative hypothesis, the test statistic, and the *P*-value.

Indicate the alternate hypothesis





Testing Claims about a Population Mean μ

A test prep company claims a mean improvement of 50 points on a standardized test. (n=40)

Show all eight steps. Then repeat using TTEST.

(Step 6: x-bar=47, s=8.6)

Testing Claims about a Population Proportion *p*

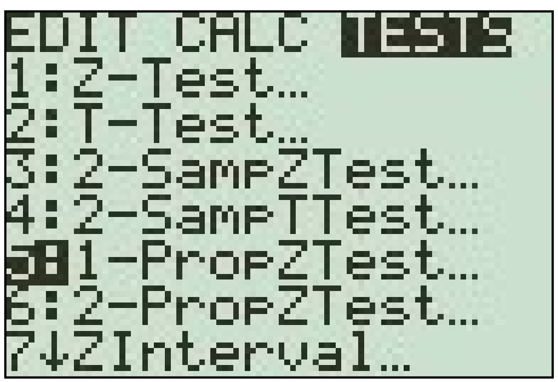
A pharmaceutical company claims its drug is effective on 80% of patients. (n=200)

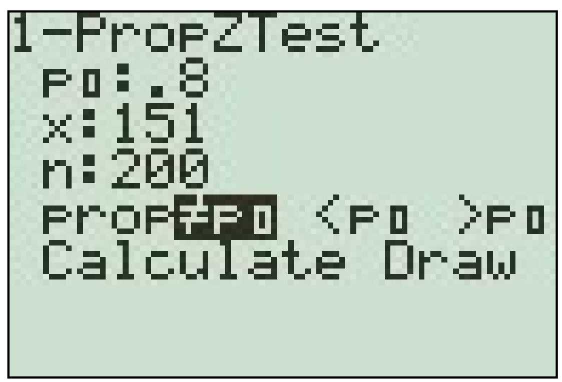
(One could argue that this should be a one-tailed test – and that is what I would choose – but I will treat this as a two-tailed test for demonstration purposes.)

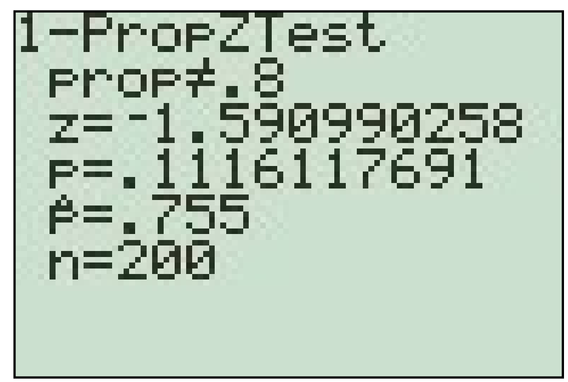
(Step 6: x=151)

Testing Claims about a Population Proportion *p*

TI-83/84 PLUS: Press **STAT**, select **TESTS**, and then select **1-PropZTest.** Enter the claimed value of the population proportion for **p0**, then enter the values for **x** and **n**, and then select the alternate hypothesis. Highlight **Calculate**, and then press the **ENTER** key.







Appendix

SPSS Output

	Test Value = 5					
					95% Confidence Interval of the Difference	
	t	df	Sig (2-tailed)	Mean difference	Lower	Upper
AGE	-1.094	26	.284	-.4000007	*******	*******

"Sig" is the p-value.

Testing Claims about a Mean: σ known

TI-83/84 PLUS: If using a TI-83/84 Plus calculator, press **STAT**, then selected **TESTS** and choose the first option, **Z-Test**. You can use the original data or the summary statistics (**Stats**) by providing the entries indicated in the window display. The first three items of the TI-83/84 Plus calculator results will include the alternative hypothesis, the test statistic, and the *P*-value.

Testing Claims about a Mean: σ known

A sample of 106 body temperatures with a mean of 98.20°F. Assume that σ is known to be 0.62°F. Consider a hypothesis test that uses a 0.05 significance level to test the claim that the mean body temperature is less than 98.6°F.

a. What is the test statistic?

b. What is the critical value?

c. What is the P-value?

d. What is the conclusion about the null hypothesis (reject, fail to reject)?

e. What is the final conclusion in simple nontechnical terms?

Tests of Hypotheses -- Eight Steps

8. Find the p-value → t test

	A	B	C
1	Mean under the null hypothesis	8	= mu0
2	Observed mean	7.8	= xbar
3	Sample standard deviation	0.4	= s
4	Sample size	18	= n
5			
6	Observed value of the test statistic	-2.1213	= t
7			
8	Ho: mu = 8 vs. H1: mu < 8	0.0244	= P(z < -2.1213)
9	Ho: mu = 8 vs. H1: mu > 8	0.9756	= P(z > -2.1213)
10	Ho: mu = 8 vs. H1: mu != 8	0.0489	= P(z < -2.1213 or z > +2.1213)

ttest ztest prop

B6: =(B2-B1)/(B3/SQRT(B4))

B8: =T.DIST(B6,B4-1,TRUE)

B9: =1-B8

B10: =MIN(B8,B9)*2

Tests of Hypotheses -- Eight Steps

8. Find the p-value → z test

	A	B	C
1	Mean under the null hypothesis	50	= mu0
2	Observed mean	47	= xbar
3	Population standard deviation	8.6	= sigma
4	Sample size	40	= n
5			
6	Observed value of the test statistic	-2.2062	= z
7			
8	Ho: mu = 50 vs. H1: mu < 50	0.0137	= P(z < -2.2062)
9	Ho: mu = 50 vs. H1: mu > 50	0.9863	= P(z > -2.2062)
10	Ho: mu = 50 vs. H1: mu != 50	0.0274	= P(z < -2.2062 or z > +2.2062)

ttest ztest prop

B6: =(B2-B1)/(B3/SQRT(B4))

B8: =NORM.S.DIST(B6,TRUE)

B9: =1-B8

B10: =MIN(B8,B9)*2

Tests of Hypotheses -- Eight Steps

8. Find the p-value → Prop z test

	A	B	C
1	Trials	200	= n
2	Successes	151	= x
3	Hypothesized proportion	80%	= p0
4			
5	Sample proportion	0.755	= pHat
6	Observed value of the test statistic	-1.5910	= z
7			
8	Ho: p = 80% vs. H1: p < 80%	0.0558	= P(z < -1.591)
9	Ho: p = 80% vs. H1: p > 80%	0.9442	= P(z > -1.591)
10	Ho: p = 80% vs. H1: p != 80%	0.1116	= P(z < -1.591 or z > +1.591)

ttest | ztest | prop

B6: =(B5-B3)/SQRT(B3*(1-B3)/B1)

B8: =NORM.S.DIST(B6,TRUE)

B9: =1-B8

B10: =MIN(B8,B9)*2

(We will do this next week for review)

Your company has purchased a machine with which it expects to produce six items per day. The production output for a sample of seven days is as follows:
3, 7, 7, 3, 4, 8, 3.

a) Find the sample mean. Does the machine appear to be producing as expected?

b) Conduct a two-tailed test of hypothesis that the mean output is six items per day. Does the machine appear to be producing as expected?